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# Broken Solitons Skyrmions

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The problem of constructing internally rotating solitons of fixed angular frequency $\omega$ in the family of Skyrme models is reformulated as a variational problem for an energy-like functional, called pseudoenergy, which depends parametrically on $\omega$. Our results confirm the existence of two types of instabilities determined by the relation between the mass parameter $\mu$ of the potential and the angular frequency $\omega$.

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Many field theories of interest in fundamental physics support topological solitons – spatially localized, stable lumps of energy whose strongly particle-like characteristics make them natural theoretical models of elementary particles. Perhaps the best developed model from this viewpoint is the Skyrme model, whose solitons are posited to model atomic nuclei.

Together with the original Skyrme model in $d = 3 + 1$ [1] and the Faddeev–Skyrme model [2], the baby Skyrme model [3, 4] can be considered as a member of the Skyrme family, the Lagrangian of all these models has a similar structure, it includes the usual $O(3)$ sigma model kinetic term, the Skyrme term, which is quartic in derivatives, and the potential term which does not contain the derivatives.

It is important that the soliton configurations possess both rotational and *isorotational* degrees of freedom. Traditional approach to study the spinning solitons is related with rigid body approximation [5, 6]. The assumption is that the solitons could rotate without changing its shape. This restriction can be weakly relaxed by consideration of the radial deformations which would not violated the rotational symmetry of the hedgehog configuration [6, 7]. Evidently, this approximation is not very satisfactory, a consistent approach is to solve a full system of field equations without imposing any spatial symmetries on the isospinning solitons. Furthermore, almost all previous studies of spinning solitons (see e.g. [8, 9]) were concerned with minimization of the total energy functional $E_J[\varphi]$ for a fixed value of the isospin $J$. However if we do not assume the spinning soliton will have precisely the same shape as the static one, this approach becomes rather involved, it is related with a numerical solution of a complicated differential-integral equation.

Recently the isospinning solitons were considered in the Faddeev—Skyrme model [8, 10] and in the baby Skyrme model [11] beyond the rigid body approximation. The approach of the papers [10, 11] is to consider the static pseudo-energy minimization problem, where the pseudo-energy functional $F_\omega[\varphi]$ depends parametrically on the angular frequency $\omega$. The important conclusion which is general for all models of the Skyrme family, is that there is a new type of instability of the solitons due to extra nonlinear velocity dependence generated by the Skyrme term [10]. Interestingly, we observe that the critical behavior of the isospinning baby Skyrmions depends also on the structure of the potential of the model, for example the isospinning configurations of higher degree may become unstable with respect to decay into constituents.

Since isorotation involves only rotational

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symmetry of the target space $S^2$, it is convenient to consider the Faddeev–Skyrme model on a general oriented Riemannian manifold $M$. This allows one to treat in unified fashion the case of principal interest, $M = R^3$, and the cases of soliton chains or strings, $M = R^2 \times S^1$, sheets $M = R \times T^2$, or geometrically nontrivial domains (of potential interest for cosmological applications, for example). Given a time-dependent field $\varphi : R \times M \to S^2$, we have at each fixed time $t$ a mapping $\varphi(t, \cdot) : M \to S^2$ which we shall, in a slight abuse of notation, again denote $\varphi$, and a time derivative $\dot{\varphi}$, which is a section of the bundle $\varphi^{-1}TS^2$ over $M$. Using these, we define, at time $t$, the kinetic and potential energy functionals to be

$$T = \int_M \frac{1}{2}|\dot{\varphi}|^2 + \frac{1}{2}|\varphi^*(\iota_{\dot{\varphi}}\Omega)|^2; \qquad V = \int_M \frac{1}{2}|\mathrm{d}\varphi|^2 + \frac{1}{2}|\varphi^*\Omega|^2 + U(\varphi), \tag{1}$$

where $\Omega$ is the area form on $S^2$, $\varphi^*\Omega$ its pullback to $M$, $\iota$ denotes interior product, and $U : S^2 \to [0, \infty)$ is a smooth potential function which we assume attains its minimum value 0 at some point $\psi_\infty \in S^2$, and is invariant under rotations about $\psi_\infty$. If $M$ is noncompact, we assume that $\varphi(t, x) \to \psi_\infty$, as $x \to \partial_\infty M$, sufficiently fast for all integrals to converge.

A uniformly isorotating field $\varphi(t, x) = R(\omega t)\ \psi(x)$ is a critical point of the restricted action $S : X_\omega^{S^1} \to R$ iff the static field $\psi : M \to S^2$ is a critical point of the pseudoenergy functional

$$F_\omega(\psi) = \int_M \left\{ \frac{1}{2}(|\mathrm{d}\psi|^2 - \omega^2|\mathrm{d}(\psi_\infty \cdot \psi)|^2) + \frac{1}{2}|\psi^*\Omega|^2 + (U(\psi) - \frac{1}{2}\omega^2|\psi_\infty \times \psi|^2) \right\}. \tag{2}$$

The first two terms of (2), taken together, can be interpreted as the Dirichlet energy of the map $\psi : M \to S^2$, where $S^2$ is given the deformed metric

$$\langle X, Y\rangle_\omega = X \cdot Y - \omega^2(\psi_\infty \cdot X)(\psi_\infty \cdot Y) \tag{3}$$

for all $X, Y \in T_\psi S^2$. For $0 < \omega < 1$ this metric gives $S^2$ the geometry of an oblate sphere, squashed along the direction of $\psi_\infty$. For $\omega > 1$, the metric is singular, changing from Riemannian to Lorentzian in a strip around the equator (orthogonal to $\psi_\infty$). Consequently, the pseudoenergy $F_\omega$ is no longer bounded below for $\omega > \omega_1 = 1$ which has strong consequences. The fourth and fifth terms together can be interpreted as a deformed potential

$$U_\omega(\psi) = U(\psi) - \frac{1}{2}\omega^2|\psi_\infty \times \psi|^2. \tag{4}$$

Since $U$ attains its minimum at $\psi_\infty$ and is rotationally invariant, its hessian about $\psi_\infty$ must be $\mu^2\langle\cdot,\cdot\rangle$ for some constant $\mu \geq 0$, interpreted physically as the mass of mesons in the field theory. Hence, if $\omega > \omega_2 = \mu$, $F_\omega$ is again unbounded below.

Thus, there is no reason why isospinning solitons should persist for frequencies $\omega \in (1, \mu)$, since $F_\omega$ is unbounded below when $\omega > \min\{1, \mu\}$. Hence, we have the possibility that isospinning hopfions are destabilized by nonlinear velocity terms in the field equation *before* they reach the upper limit $\omega = \mu$.

Full scale numerical calculations performed in our works [10, 11] confirmed this conclusion, both in the Faddeev–Skyrme and the baby-Skyrme model.

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